

Design of a Reinforced  
Concrete Steel Arch Bridge

Stanley Dean  
J. C. Penn

1905

624.6  
D 34

ARMOUR  
INST. OF TECH. LIB.  
CHICAGO.



**Illinois Institute  
of Technology  
Libraries**

AT 6

Dean, Stanley.

Design of a reinforced  
concrete steel arch bridge







Design of a Reinforced  
CONCRETE STEEL ARCH BRIDGE

A Thesis presented by

STANLEY DEAN & JOHN C. PENN

to the

President & Faculty

of the

Armour Inst. of Technology

for the Degree

of

Bachelor of Science in Civil Engineering

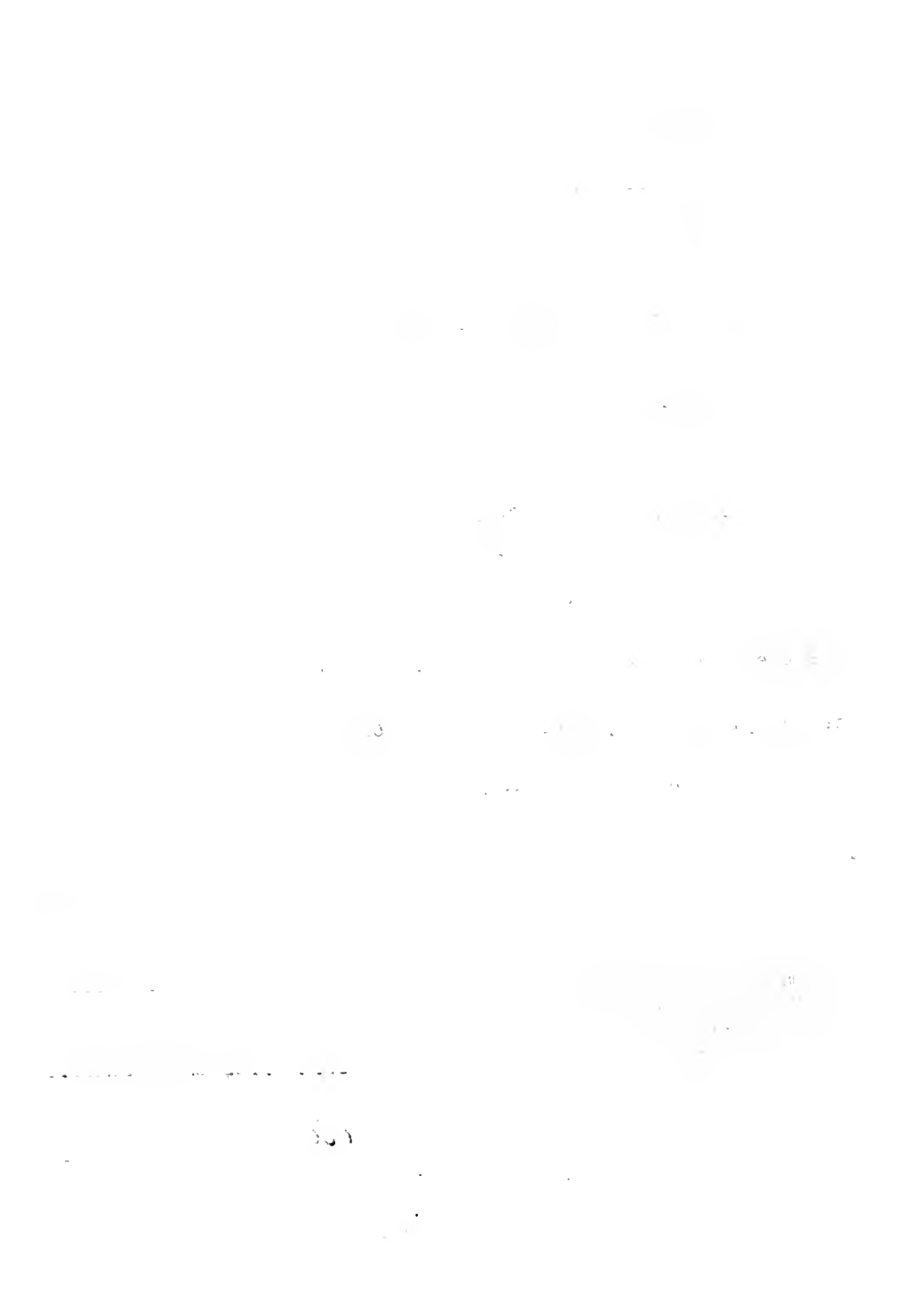
Having completed the prescribed course of study in

Civil Engineering.

Chicago, June 1905.

ILLINOIS INSTITUTE OF TECHNOLOGY  
PAUL V. GALVIN LIBRARY  
35 WEST 33RD STREET  
CHICAGO, IL 60616

*L. M. Raymond*  
Dean of Engineering  
*J. C. Monin*  
Dean of Cultural Affairs  
*Wm. E. Kellogg*  
Dep. Cent. Engineering





## The Design of a Reinforced Concrete Arch Bridge.

### Data:--

The design of this bridge was made to cover an actual case, with the exception of its length. In the actual case five spans of 50 ft. each and two of 40 ft. would be necessary, while for the purpose of this design, one span of 50 ft. and 2 of 40 ft., were chosen. The piers and abutments are set on a solid limestone rock foundation. The Elevation of the crown of the road at the ends is 19.15 ft.; the bottom of river is 4.5 ft., and practically level. The assumed elevation of springing <sup>n</sup>live is 8.5 ft. Width of road was assumed as 24 ft.; together with two sidewalks of 6 ft. each, gives a total width of Roadway of 36 ft. The Roadway was given a grade of 5% from the ends towards the center.

In the following discussion all figures will refer to the 50 ft. span arch. The design of this arch will be given in detail, while merely the figures for the 40 ft. span will be given.

### Vertical Dead Load:--

The weight of concrete was taken as 130 # per cu. ft., the earth filling as 100 # per cu. ft. The factor of safety for the dead load is four (4).

The line of stress of the arch was assumed to be <sup>of</sup>parabolic form. For the 50 ft. arch, a rise of 8 ft. was assumed. From existing structures and plans made by the St. Louis Expanded Metal Company, a rough plan of the arch ring was drawn, thickness

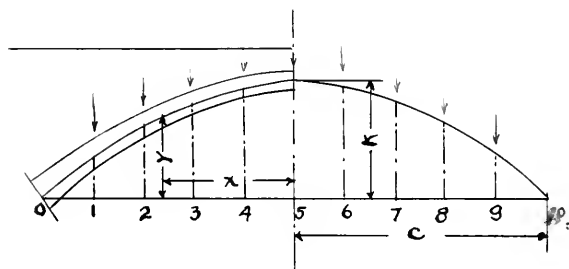
100

100

100

100

of crown as 16 in., of haunches 24 in. By scale, the actual dead load of the arch was computed. For figuring the stresses, the loads were assumed as applied at 10 panel points numbered as shown in sketch. In table "concrete area" is the area of the



arch ring at the panel point.

This multiplied by the 130,

gives the weight of the concrete applied at that point, the arch ring being assumed to be one foot wide.

Likewise the area of filling was scaled, and this multiplied by 100 gives the weight of filling. The sum of these two X the factor of safety of 4, gives the Dead panel load for that point.

#### Vertical Live Load:--

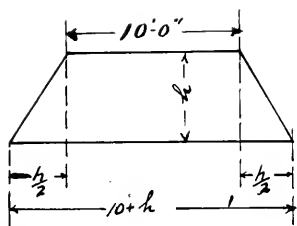
A concentrated load of a 20 ton Road Roller, on 2 arches, 5 ft. long and 10 ft. between centers, was used to determine an equivalent uniformly distributed live load.

This load of 40,000 lbs. can be considered as distributed over 20 ft. of roadway or equals 2000 lbs. per lineal foot of Roadway. The axles being 5 ft. long, the broad tires of the roadroller would further distribute the load over about 10 ft. of width, giving a load of 200 # per lineal feet of arch one foot wide.

The actual pressure on the arch rib is much less than at



the crown i.e. the earth filling further distributes the Live Load. The angle of repose being more nearly vertical in all



probability than ordinary earth, in equilibrium, say  $1/2$  to  $1$ , the 2000 # distributed over 10 Ft., at the surface at a depth of  $h$ , below the surface would be dis-

tributed over  $10+h$  feet. The load per sq. foot of arch then is  $\frac{2000 \#}{10+h}$  and with a factor of safety of eight (8) gives the Live Load per sq. foot as  $\frac{16000}{10+h}$ . The Live Load per panel point, panels being 5 ft.,  $= \frac{80,000}{10+h} \cdot h$ . 'h' was measured from the sketch and the Live Load for the different points calculated. (See table).

#### Horizontal Dead Load:--

According to Rankin's earthwork formulae, the horizontal intensity is to that of a vertical load as  $1:3$  when angle of surface  $= 0^\circ$  and angle of repose  $= 30^\circ$ .

However the horizontal pressure acts on smaller surface than the corresponding vertical load i.e., if  $p$  = panel length,  $h$  = vertical distance between adjacent panel points.  $P$  = vertical load and  $h$  = horizontal load,  $h = \frac{Ph}{3p}$ . The Horizontal Dead Loads were accordingly calculated.

#### Horizontal Live Load:--

Was calculated in a similar manner.

#### Stresses:--

The stresses in the arch rib, due to bending moment,



thrust, and shear were figured according to Prof. Greene's method as given in his book, "Trusses and Arches" Part 113, Pages 60 to 62 & 116 to 119. The arch rib is considered to be of Parabolic shape with fixed ends. The actual tables for calculating the bending moments, thrusts and shear, were taken from Walter W. Colpitts' book on the "Calculation of the Stresses and Practical Design of Structures of Steel Concrete." The figures in the tables as given in the latter are merely those of Greene multiplied by twelve, to reduce the bending moment to inch pounds. It was not considered necessary to reproduce these tables as the ~~se~~<sup>se</sup> constants can be found in text books and back numbers of the Engineering News.

The Actual figures however are given in the following tables:-

Bending Moment. Vertical Live Load Table

Horizontal "	"	"
Vertical Dead	"	"
Horizontal "	"	"
Thrust	"	"
Shear	"	"

Temperature;--

Again according to Prof. Greene, the Bending Moment at the crown due to a change of temperature =

$$\frac{15}{4} \times \frac{t e E I}{12 k}$$

If  $t = 75^{\circ} \text{F.}$ , and  $e = \text{Coeff. of expansion of concrete} = .0000055$  per degree  $\text{F.}$ ,  $E = \text{Mod. of Elasticity of concrete} = 3,000,000$  pounds per square inch, and  $I = \text{Moment of Inertia of Section at crown}$

1. The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \int_0^x \frac{1}{1+t^2} dt.$$

It is shown that the function  $f(x)$  is increasing and concave down on the interval  $(-\infty, \infty)$ . Moreover, the function  $f(x)$  is bounded on the interval  $(-\infty, \infty)$ .

2. In the second part of the paper, we study the properties of the function  $g(x)$  defined by the equation

$$g(x) = \int_0^x \frac{1}{1+t^4} dt.$$

It is shown that the function  $g(x)$  is increasing and concave down on the interval  $(-\infty, \infty)$ . Moreover, the function  $g(x)$  is bounded on the interval  $(-\infty, \infty)$ .

3. In the third part of the paper, we study the properties of the function  $h(x)$  defined by the equation

$$h(x) = \int_0^x \frac{1}{1+t^6} dt.$$

It is shown that the function  $h(x)$  is increasing and concave down on the interval  $(-\infty, \infty)$ . Moreover, the function  $h(x)$  is bounded on the interval  $(-\infty, \infty)$ .

4. In the fourth part of the paper, we study the properties of the function  $k(x)$  defined by the equation

$$k(x) = \int_0^x \frac{1}{1+t^8} dt.$$

It is shown that the function  $k(x)$  is increasing and concave down on the interval  $(-\infty, \infty)$ . Moreover, the function  $k(x)$  is bounded on the interval  $(-\infty, \infty)$ .

5. In the fifth part of the paper, we study the properties of the function  $l(x)$  defined by the equation

$$l(x) = \int_0^x \frac{1}{1+t^{10}} dt.$$

It is shown that the function  $l(x)$  is increasing and concave down on the interval  $(-\infty, \infty)$ . Moreover, the function  $l(x)$  is bounded on the interval  $(-\infty, \infty)$ .

6. In the sixth part of the paper, we study the properties of the function  $m(x)$  defined by the equation

$$m(x) = \int_0^x \frac{1}{1+t^{12}} dt.$$

It is shown that the function  $m(x)$  is increasing and concave down on the interval  $(-\infty, \infty)$ . Moreover, the function  $m(x)$  is bounded on the interval  $(-\infty, \infty)$ .



$$\text{then } \frac{M}{t} = 387 \frac{I}{K}$$

At the springing point, the bending moment is twice that at the crown or  $774 \frac{I}{K}$

The Horizontal thrust under the same conditions  $96 \frac{I}{K}$ .  
Shear due to a change of temperature was considered to be so small as to be negligible.

By multiplying the bending moment at the crown by the following factors, the bending moment at the respective panel point may be obtained.

Panel Points	Abut.	1	2	3	4	5
Factors	..... 2	.92	.08	.52	.88	$\times 1.0$

These factors are obtained by assuming a uniformly distributed load as having the same effect as that due to a change of temperature. The bending moment at each panel point due to a uniform load, can be calculated from the tables in terms of the load, and then a ratio established between the moments at the various points and that at the crown.

It is necessary to have a value of the moment of inertia at the crown. Consider the formula for Bending Moment as deduced in Mechanics  $M = \frac{Fy}{y}$ .  
Where F = tension or compression in outer fibre and y. is the distance of that fibre to the neutral axis, and I the moment of inertia then  $I = \frac{My}{F}$

Then finding the stress  $F_c$  due to the maximum moment, resulting from dead and live loads and calculating the distance

200

2.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

...and the

• 2000 •

y., by methods hereafter to be explained,  $I$  can be obtained, and consequently bending moment and thrust due to temperature.

In the design of sections Prof. Hatt's theory was adopted as being the most easily deduced and rational. The deduction of his formulae are given in Engineering News. Vol. 47. P. 170 and a brief outline of it follows.

Theory of the Strength of Beams of Reinforced Concrete  
by Prof. W. Kendrick Hatt.

Engineering News. Vol. 47 P. 170

An arch ring is considered as a beam in which each face may be <sup>in</sup> tension.

A. Steel - Modulus of Elasticity =  $E = 30,000,000$  lbs.

Elastic Limit =  $f = 30,000$  lbs.

B. Concrete in compression #, Mod. of Elasticity

$E_c = 2,400,000$  lbs.

Compressive strength =  $c = 2000$  lbs. = maximum

C Concrete in tension strength =  $\frac{1}{10}$  of  $c = 200$  lbs.

Computation:--

Material like concrete is not perfectly elastic. The stress-strain curve is not therefore ~~xxxxx~~ a straight line up to the elastic limit and Hooke's Law cannot be assumed to hold.



The problem is to compute the ultimate or breaking load of a beam.

Assumptions:--The cross sections of the beam remain plane surfaces during flexure.

2. The applied forces <sup>are</sup> and perpendicular to the neutral surface of the beam.

3. It is assumed that the pressure of the material surrounding any elementary fibre will not modify the effect of the stress on the fibre but that the latter will elongate or be compressed just as if it were under that load by itself in a testing machine.

4. There is no slipping between the faces of the wire and the surrounding concrete.

5. The elastic limit of the concrete is exceeded in both the compression and tension flanges.

6. There are no initial stresses due to shrinkage or expansion, of the concrete while setting.

The stress-strain diagram may be used to represent the Law of increase of Stress as we go from the neutral axis outward.

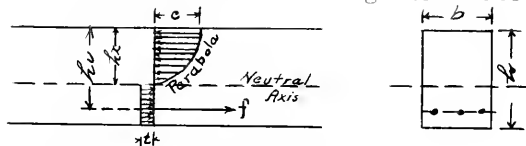


Fig. 3. illustrates the state of stress as adopted by Prof. Hatt.

Ordinarily there are given the following quantities  
Moduli of steel and concrete tensile and compressive, strength  
of concrete, size of beam and reinforcement with location of  
latter.

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

There are three conditions to be determined:

1st, the distance of the neutral axis from the upper face.

2nd, proportion of steel

3rd, moment of resistance

These can be determined by the algebraic statement of three facts; first, the total force of compression on compressive side equals the total force of tension on the tension side. Second, the extension of steel and the compression of concrete at the fibre will be to each other as the distance of these materials from the neutral axis. 3rd, the moment of external forces on one side of the section about the neutral axis = the moment of resisting stresses on the section about the same axis.

Prof. Hatt assumes the curve of compression to be a parabola and the tensional stress diagram a straight line parallel to the section.

$H$  = depth of beam

$b$  = width of beam

$h_x$  = distance of neutral axis from top of beam

$F$  = area of concrete

$F'$  = area of steel

$p = \frac{F'}{F} = \frac{1}{75}$  to  $\frac{1}{100}$  generally

$x, u$  &  $p$  are ratios.

$E_c$  = Mod. of Elasticity of Concrete.

$E_s$  = Mod. of Elasticity of Steel

$f_s$  = stress of steel

$c_c$  = compressive stress in outer fibre of concrete

$t$  = tensional " " " " "

Area in compression =  $\frac{2}{3} c_c h_x$ .





Total force of compression =  $2/3$  c.h.x.b.

Moment of compression about neutral axis =  $2/3$  c.h.x.b. $\cdot$  $\frac{2}{3}$  h,x. (~~2/3 c.h.x.b. 5/8 HX~~)

Total force of tension on concrete =  $t, h, (1-x)b \cdot$

Moment of tension about the neutral axis =  $t, h (1-x)b, h$

$$\left(\frac{1-x}{2}\right)$$

Tension on steel =  $F'f = p h b f$

Moment of this tension on steel about the neutral axis =  $p h b \cdot f h (u-x)$

From fact(1) previously referred to, we have  $2/3$  c h x b =  $t h (1-x) b + p h b f$ , or  $2/3$  c x =  $t (1-x) + p f$  (1)

From Fact 3.

$M = 2/3$  c h x b,  $5/8$  h x +  $t h (1-x) b h \left(\frac{1-x}{2}\right) + p b h f h (u-x)$

$$M = b h^2 \left\{ \frac{2}{3} c \left(\frac{1-x}{2}\right) + \frac{5}{12} c x + p f (u-x) \right\} \quad (2)$$

Let  $e_c$  = compression of fibres by compression and  $e_s$  = lengthening of fibres by tension of steel, then from definitions of  $E, E_c = \frac{c}{e_c}$

$$\text{and } \frac{E}{E_s} = \frac{f}{e_s} \left\{ \begin{array}{l} \sim E_s = \frac{f}{e_s} \\ E_c = \frac{c}{e_c} \end{array} \right.$$

From fact (2)

$$\frac{e_c}{E_c} = \frac{h x}{h(u-x)} \quad \text{or } \frac{c}{f} = \frac{E_c}{E_s} \cdot \frac{x}{u-x} \quad (3)$$

Eliminate  $f$  from (1) & (3)

$$t(1-x) + p c \frac{E_s}{E_c} \frac{u-x}{x} = \frac{2}{3} c x. \quad (4)$$

Having assumed  $c, t, E_c$  &  $E_s$  as well as  $u$  and  $f$ , from equation (3) value of  $x$  can be computed. With this value of  $x$ ,  $p$  can be computed from (4). Then with these values, having given your moment of external forces, the value of  $h$  can be found from (2).



By this method a table was made giving the value of  $M$ ,  $x$  and  $p$  for different values of  $c$ .

In the computation  $u$  was assumed as  $7/8$ . This gives a 2 inch covering for the steel in a 16 inch. beam. This value is a little low, but it allows for slight irregularities in placing the rods by the workmen. A rod may be bent slightly and if not straightened out may be at a greater distance from the face than 1 inch or  $1\frac{1}{2}$  as ordinarily allowed.

#### Design of Section of the Crown.

The design of the sections at various panel points varies very little and as an example, the complete work for the section at the crown follows:

The vertical dead load moment at the crown = -170564 In. lbs. The horizontal dead load moment = -16728 inch.lbs. Live Loads, vertical, 1-2-3-7-8-9 on the arch give a maximum negative bending moment of -194136 inch lbs. Horizontal Live Loads placed at these points give a moment of -12314 in.lbs.

Loads placed at 4-5-6 give a maximum positive moment of +89132 and horizontal loads placed at these points give a moment of -1284 inch lbs. Live loads producing positive moment give a total moment for the crown equal to -99444 inch.lbs. Live loads producing negative moment, give a total negative moment of 393742 inch.lbs. The negative moment of the dead load always overbalances the positive moment of the live load and consequently no positive moment at the crown.

The horizontal thrust due to the loads giving a moment of -393742 equals 63280 lbs.

Loads giving maximum moment usually also give the maximum thrust.



As stated before, it is necessary to design a temporary value for the section in order to find a value for the moment of inertia.

Assuming an allowed value of 1750 lbs. per sq. inch on the concrete to resist the bending moment from the designing table

$$M \text{ equals } 3488 \frac{h^2}{2}$$

$$\text{or } h = \sqrt{\frac{M}{3488}}$$

$$M \text{ equals } 393742$$

$$h = 10.6 \text{ "}$$

and the area of the section equals  $12 \times 10.6$  or  $127.2 \text{ "}$

The thrust per square inch on the section then equals

$$\frac{63480}{1272} \text{ equals } 500 \text{ \#}$$

The total allowed thrust for concrete equals 2000 #

The direct thrust taking 500 # of this, the amt. left. i.e., 2000-500 or 1500 # can be utilized for bending moment. A new value of c. must therefor be chosen.

$$\text{Let } c \text{ equal } 1500, \text{ then } h \text{ equals } \sqrt{\frac{393742}{2666}} \text{ equals } 12.2 \text{ "}$$

This gives an area of  $12.2 \times 12$  equals 1464 sq. inch. The thrust then per square inch equals  $\frac{63480}{1464}$  equals 433 lbs. 1500 plus 433, equals 1933 lbs. total compression on concrete, well within the limit of 2000 #

For this value of c, x equals .336 h or 4.06

$$\text{The moment of inertia equals } \frac{M \times y}{c} \text{ equals } \frac{393742 \times 4.06}{1500}$$

equals 1068.

The moment at the crown due to temperature equals  $337 \frac{I}{K}$



equals 51700 " lbs.

The horizontal thrust equals  $\frac{996}{K_2}$  equals 1602 lbs.

The total negative moment therefor equals  $-393742 + 51700$  equals  $-415442$ . The minimum moment equals  $-99444$  plus  $+51700$  equals  $-47744$ , since the moment due to temperature can be either negative or positive accordingly as there is a rise or fall in temperature, i.e., the moment must be added arithmetically to the moment due to the actual loads.

The total horizontal thrust equals 63280 plus 1602 equals 64882 lbs.

These are the final figures for the design of the section

Assumed  $c$  equals 1500 #

$$h = \sqrt{\frac{445442}{2682}} \text{ equals } 12.87 "$$

Area of section equals  $12 \times 12.87$  equals 154.24 sq. inches

$$\text{Pressure } = \frac{64882}{154.24} \text{ equals } 418. - 10.$$

The total compression equals 1500 plus 418 # equals 1918 #

Interpolating for a new value of  $c$ .

2000-1918 equals 82. Let  $c$  equal 1500 plus 82 equals 1582 lbs. For this value of  $c$ , by interpolation,  $M$  equals 2953 <sup>2</sup> h

$$h \text{ equals } \sqrt{\frac{445442}{2953}} \text{ equals } 12.28 "$$

$$\text{Pressure equals } \frac{64882}{12.28 \times 12} \text{ equals } 437.5$$

and total compression equals 1582 plus 437.5 equals 2019.5

This value of total compression is a little too large and a new calculation might be gone through out for all practical purposes

$h$  may be assumed as 12.3"  $x$ , For  $c$  equals 1582, is equal to

.347  $h$  or 4:27 ". Value of  $p$  for  $c$  equals 1582 equals .00778

or the area of steel for the section equals .00778  $\times 12 \times 12.28$  equals 1.136 sq. inch.

0

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

8

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974

1. 10. 1974



There being no positive moment, this completes the design of the section for the crown.

An area of steel equals 1.136 sq. inches for a width of 12 equals practically  $7/8$  " round rods spaced at  $5 \frac{1}{2}$  " center to center.

The sections at the other panel points are calculated in a similar manner.

When the values of  $x$  and  $h$  have been obtained, for both positive and negative bending moments, for every panel point, the values of  $h-x$  are laid off to scale above the parabola for **negative** moment and below the parabola for positive moment at the proper panel points. A smooth curve drawn through these points, one above and one below the parabola give the **extradosal** and **intradosal** lines of the arch. Under no circumstances may **these** lines go within the plotted points. At the haunches and even at the other points, ~~xxxx~~ they are liable to fall very much outside of the points, when the area of steel may be decreased proportionally to its increased distance from the neutral axis, the parabola.

The area required for shear will be found to fall within the area required for bending moment and thrust and, consequently, no reinforcement is required for this. However to prevent any tendency of the steel bars on the lower side from straightening and consequently tearing out of the concrete when stretched, it is well to put in inclined bars, inclined about  $45^\circ$ , hooking the lower bars to the upper bars and thus preventing any such tendency. These bars will also take up any shearing stresses that may not have been prepared for.



Piers and abutments were designed to conform with the three following conditions.

1st: Area of base must be sufficient to give a unit pressure on both concrete and rock foundation, less than the crushing strength of concrete.

2nd: Line of stress, due to eccentric loading must fall within the middle third of the base.

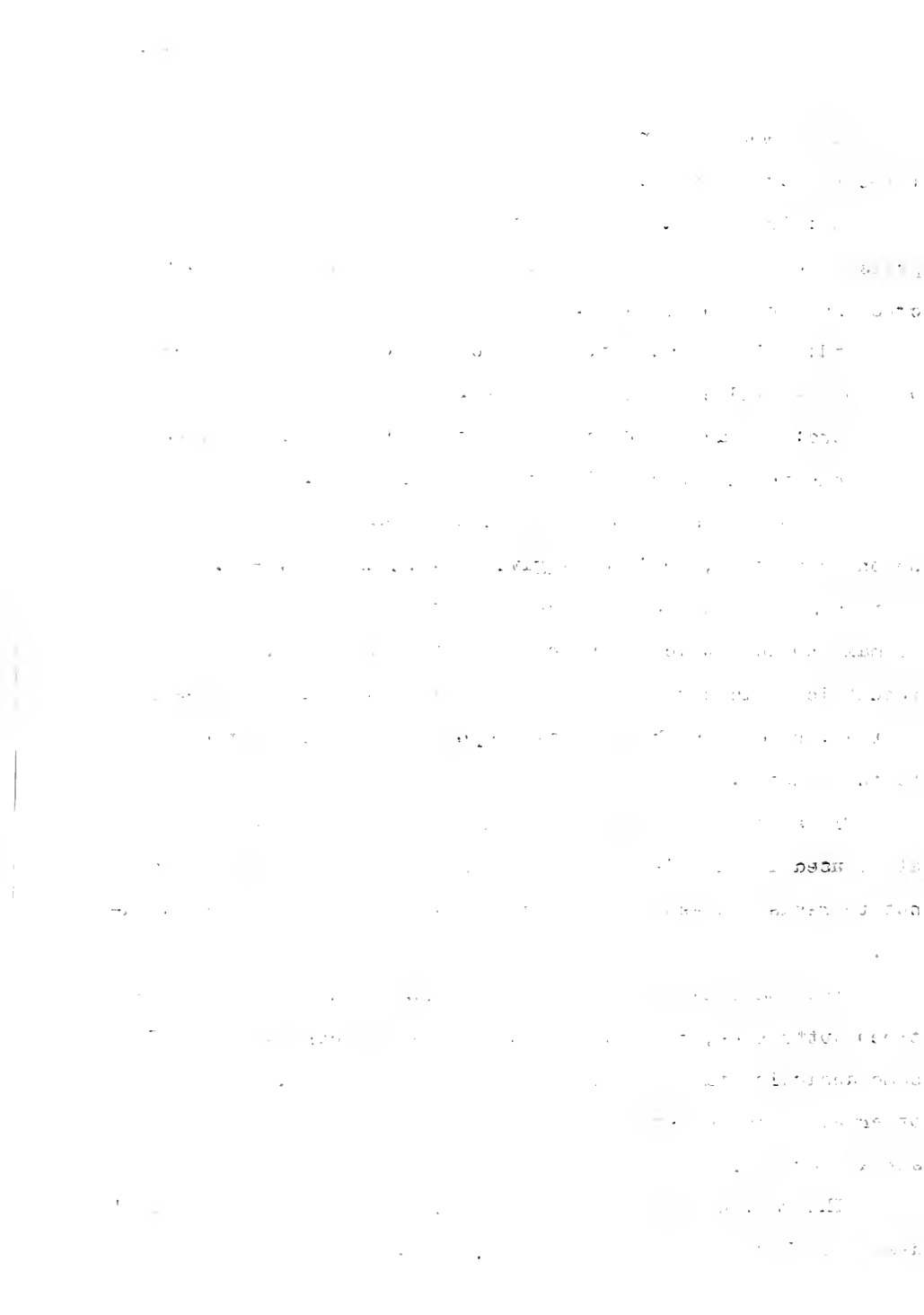
3rd: The angle of the resultant with the vertical, must be less than the angle of friction of the material.

The method used for first is obvious. To satisfy the second condition, the loading giving max.  $\downarrow$  and max.  $- M.$  at the abutments was considered since this loading gives the condition of maximum eccentricity and consequent instability. The eccentricity is equal to the bending moment divided by resultant of the vertical and horizontal components of the reaction due to the loading.

Piers and abutments were figured graphically and all steel used in them is merely for bending and temperature and not to resist stresses that may come into them due to the loading.

The Spandrel Walls were figured as horizontal beams between buttresses, the latter being figured as cantilevers. The same designing tables were used as those for the arch rib. All other wires in the structure are for the purpose of preventing surface cracks.

The railing and posts were copied from the Wabash R.R. Co's design of Forest Park Bridge at St. Louis.



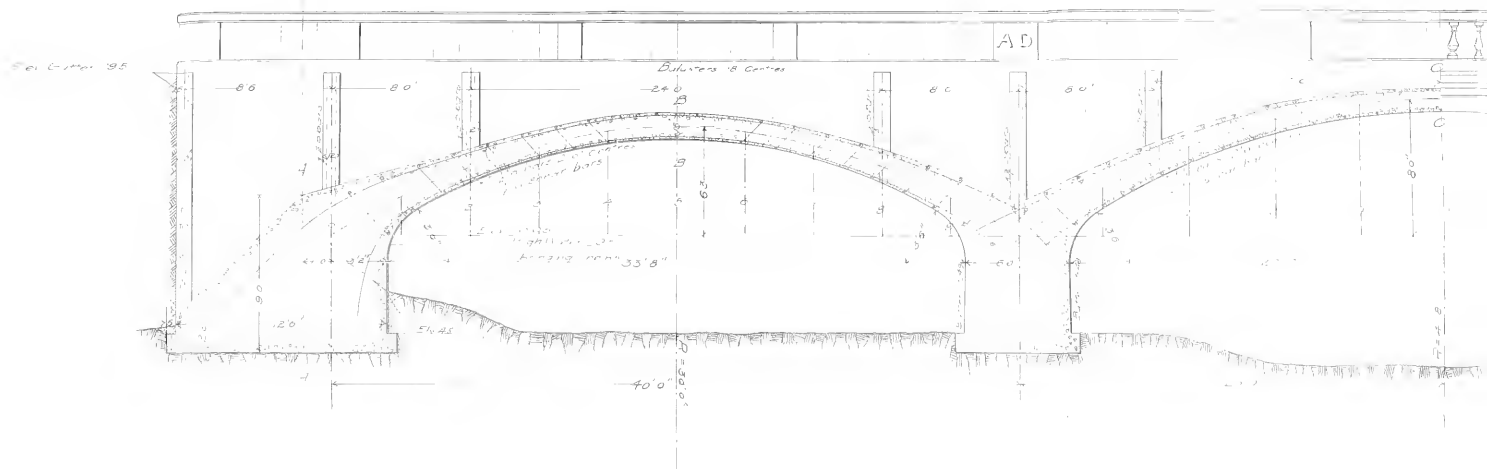
In the design of Reinforced Concrete Arch many assumptions must be made. Prof. Greene makes assumptions in his theory of stresses in an arch. Prof Hatts ~~makes~~ assumptions in his theory of Resistance of Beams to flexure. Allowed stresses are assumed. large factors of safety or ignorance are used to take care of our lack of knowledge in regard to reinforced concrete. Notwithstanding all this until we have a fuller knowledge of Reinforced concrete (and this present Reinforced Concrete fad will bring it out), this method gives us a means of designing Concrete Arches economically, Arches that will hold the load for <sup>such</sup> ~~that~~ they are designed and yet have no more material in them than seems necessary.

*John C.*



DESIGN  
ORCED-CONCRI  
MOUR INSTITUTE  
CHICAGO, ILL.

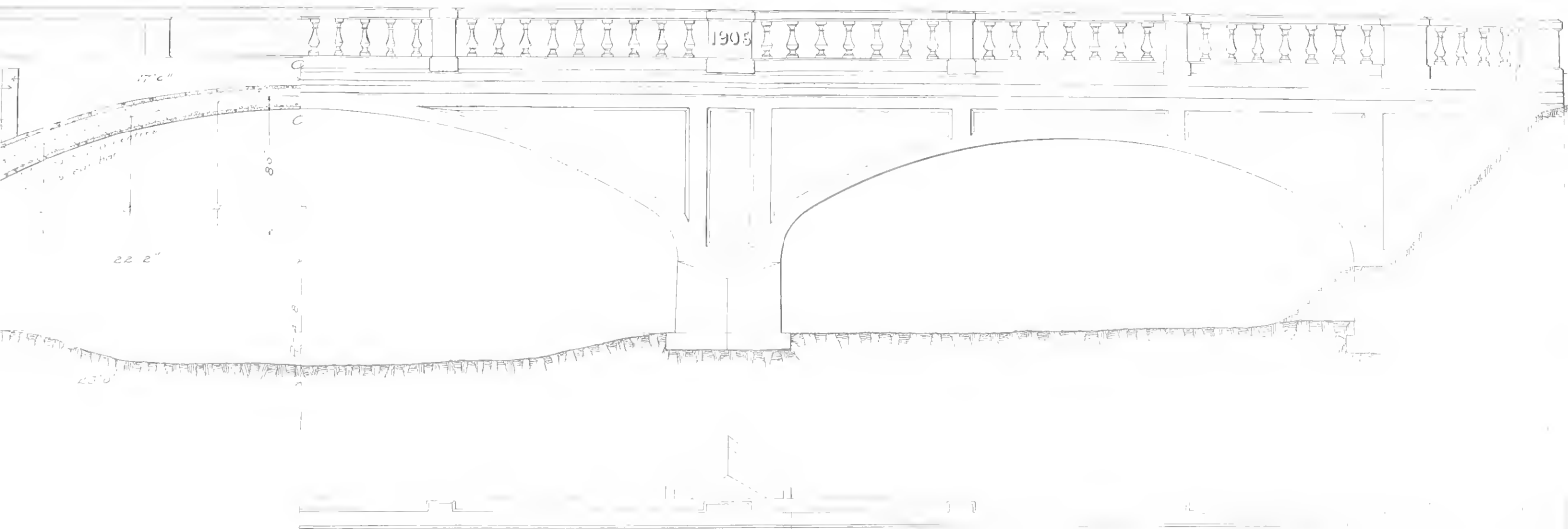
Thesis o



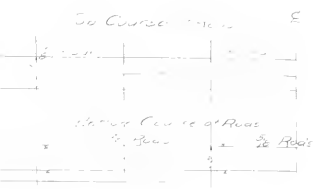
Full Section A-D

20.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 21.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 22.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 23.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 24.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 25.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 26.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 27.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 28.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 29.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 30.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$





1. Bridge spans 100 ft. over  
 2. The river, and is 10 ft. wide  
 3. at the bridge  
 4. The bridge is 10 ft. wide at the  
 5. ends.



DATA  
 Three Spans  
 Total of Main span 24 ft.  
 Total of side spans 32 ft.  
 The main span 20 ft. on Road side of 200 ft. on side  
 Designing Filling, 100 ft. on side of Arch 30 ft. on side  
 Material Compression Concrete 4500 lb. sq. ft.  
 Tensile strength 1000 lb. sq. ft.  
 The bridge is 10 ft. wide at the bridge and 10 ft. wide at the ends.  
 The bridge is 10 ft. wide at the bridge and 10 ft. wide at the ends.  
 Scale 1/4" = 1 ft.

# DESIGN OF REINFORCED-CONCRETE ARCH BRIDGE ARMOUR INSTITUTE OF TECHNOLOGY

CHICAGO, ILL. MARCH, 1903.

Thesis of {  
 Student {  
 Faculty {

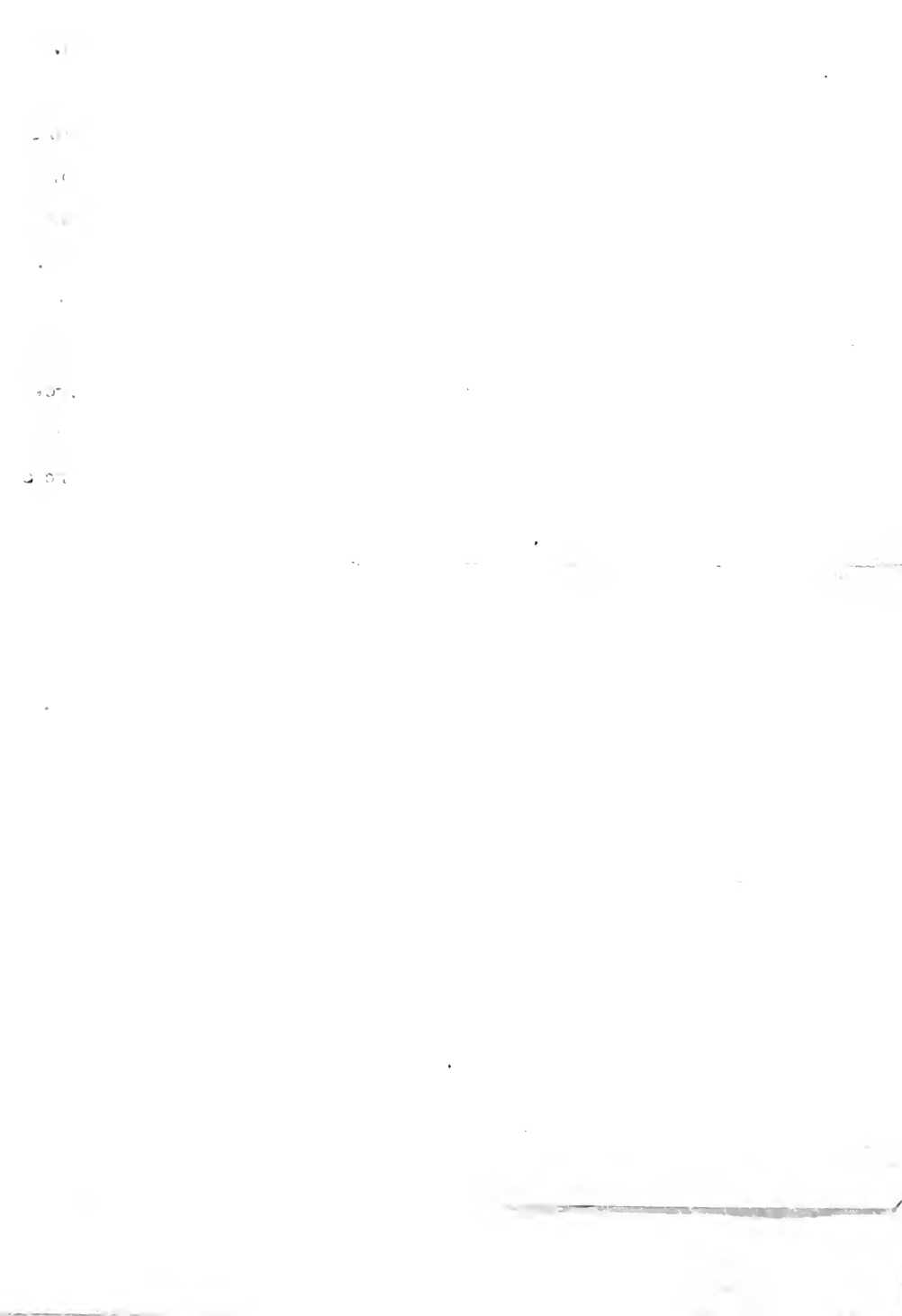


Table 1

Ordinates to Parabolic

Panel Point	0	1	2	3	4	5
Ordinate	0	$\frac{4x^2}{3l}$	$\frac{16x^2}{27l}$	$\frac{64x^2}{81l}$	$\frac{16x^2}{27l}$	$\frac{4x^2}{3l}$

$$y = \frac{4x^2}{3l} \left(1 - \frac{x^2}{l^2}\right)$$

y = Ordinate

l = total span, or rise

x = abscissa

c = half-span

Designing Table for Bending Moments

	c	M	lx	p
1	100	118.17	25.74	.0075
2	100	106.67	25.54	.0075
3	100	263.67	35.54	.0067
4	100	346.67	36.54	.0060
5	100	456.67	36.54	.0050

c = allowed compression for 6" or concrete, by 100

M = Bending moment in inch pounds

lx = distance from Neutral axis to Compression fiber of beam

p = ratio of area of steel to concrete

Vertical Reactions due to Vertical Loads

Load at	1	2	3	4	5
Live Load $\left\{ \begin{array}{l} P \\ W \end{array} \right.$	44.72	52.0	57.00	49.0	36.7
Dead Load $\left\{ \begin{array}{l} P \\ W \end{array} \right.$	1.52	2.0	1.64	2.00	2.50
Dead Load $\left\{ \begin{array}{l} P \\ W \end{array} \right.$	1.00	1.00	1.00	1.00	1.00
Dead Load $\left\{ \begin{array}{l} P \\ W \end{array} \right.$	1.00	2.00	1.00	2.00	1.00

Note-Vertical Reactions due to horizontal loads neglected as unimportant.



Table 2. Vertical Dead Load 50' Span

Panel Point	Span	Center Point	1st Point	2nd Point	Total	Span
5	5	25	30	35	105	50
4	15	35	40	45	125	40
3	25	45	50	55	150	30
2	35	55	60	65	175	20
1	45	65	70	75	200	10

Table 3 Load on 50' Arch

Panel Point	1	2	3	4	5
Live Load - Vertical	100	250	400	550	700
" " Horizontal	50	125	200	275	350
Dead Load - Vertical	100	250	400	550	700
" " Horizontal	50	125	200	275	350

Table 4 Vertical Load Components on the 100' Span

Vertical Load	1	2	3	4	5
Live Load	100	250	400	550	700
Dead Load	100	250	400	550	700

#### Horizontal Load

1st Point	$H_1 = 100$	$= 500$	$= 100$	$= 50$
" " 2nd	$H_2 = 30$	$= 150$	$= 30$	$= 15$
3rd Point	$H_3 = 100$	$= 500$	$= 100$	$= 50$
" " 4th	$H_4 = 30$	$= 150$	$= 30$	$= 15$

Note: Subsequent points are similar to the first point.



Table of Moments - 50 Feet Arch

Table 3.

## Vertical Dead Loads

<del>Mom</del> <del>Load</del> 47	0	1	2	3	4	5	6
0	+ 119,140	+ 54,221	= 27,110	= 13,555	= 6,777	= 3,389	= 1,694
3	+ 146,620	+ 61,721	= 30,860	= 15,430	= 7,715	= 3,857	= 1,929
7	+ 230,720	+ 100,720	= 50,360	= 25,180	= 12,590	= 6,295	= 3,147
6	+ 171,120	+ 72,720	= 36,360	= 18,180	= 9,090	= 4,545	= 2,273
5	+ 100,720	= 10,720	= 5,360	= 2,680	= 1,340	= 670	= 335
4	0	= 30,720	= 15,360	= 7,680	= 3,840	= 1,920	= 960
3	- 130,420	= 30,720	= 15,360	= 7,680	= 3,840	= 1,920	= 960
2	- 100,720	= 30,720	= 15,360	= 7,680	= 3,840	= 1,920	= 960
1	- 60,020	+ 27,720	+ 13,860	+ 6,930	+ 3,465	+ 1,733	+ 866
0	+ 60,020	+ 30,020	+ 15,010	+ 7,505	+ 3,753	+ 1,876	+ 938
Net H	= 49,220	= 30,860	= 15,430	= 7,715	= 3,857	= 1,929	= 964

## Horizontal Dead Loads

<del>47</del> <del>Load</del> 47	0	1	2	3	4	5	6
0	+ 7,000	+ 13,750	= 6,875	= 3,438	= 1,719	= 859	= 429
3	+ 5,500	+ 12,250	= 6,125	= 3,063	= 1,531	= 766	= 383
7	+ 4,000	+ 9,750	= 4,875	= 2,438	= 1,219	= 609	= 305
6	+ 2,500	+ 7,750	= 3,875	= 1,938	= 959	= 479	= 239
5	0	0	0	0	0	0	0
4	= 1,000	= 4,250	= 2,125	= 1,063	= 531	= 266	= 133
3	= 500	= 1,750	= 875	= 438	= 219	= 109	= 55
2	= 500	= 1,750	= 875	= 438	= 219	= 109	= 55
1	= 4,000	= 13,750	= 6,875	= 3,438	= 1,719	= 859	= 429
0	+ 14,000	+ 27,750	= 13,875	= 6,938	= 3,469	= 1,734	= 867
Net H	= 5,500	= 13,750	= 6,875	= 3,438	= 1,719	= 859	= 429





Table of Moments - Self Correction  
Vertical Live Loads

Table 4

~~Not  
Load  
1971~~

	A	B	C	D	E	F
1	+ 20,700	- 20,700	- 7,700	- 21,000	- 10,000	- 2,000
2	+ 11,200	+ 12,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
3	+ 10,700	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
4	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
5	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
6	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
7	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
8	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
9	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
10	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
11	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
12	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
13	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
14	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
15	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
16	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
17	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
18	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
19	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
20	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000

~~Not  
Load  
1971~~ Horizontal Live Loads

	A	B	C	D	E	F
1	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
2	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
3	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
4	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
5	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
6	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
7	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
8	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
9	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
10	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
11	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
12	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
13	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
14	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
15	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
16	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
17	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
18	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
19	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000
20	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000	+ 10,000

Table of Shear -  
Barriers, Joists, etc.

	A	B	C	D	E	F
Dead Load	40,000	27,700	12,000	20,000	10,000	10,000
Live Load	20,000	10,000	10,000	10,000	10,000	10,000
Horizontal Load	0	0	0	0	0	0
Temperature	0	0	0	0	0	0
Barrel Vents Shear	0	0	0	0	0	0
Shear Wall to Wall	0	0	0	0	0	0
Horizontal Shear	0	0	0	0	0	0



Table 5 Horizontal Thrust Fire Resistance Test  
 1000 lbs. at 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 210, 220, 230, 240, 250, 260, 270, 280, 290, 300, 310, 320, 330, 340, 350, 360, 370, 380, 390, 400, 410, 420, 430, 440, 450, 460, 470, 480, 490, 500, 510, 520, 530, 540, 550, 560, 570, 580, 590, 600, 610, 620, 630, 640, 650, 660, 670, 680, 690, 700, 710, 720, 730, 740, 750, 760, 770, 780, 790, 800, 810, 820, 830, 840, 850, 860, 870, 880, 890, 900, 910, 920, 930, 940, 950, 960, 970, 980, 990, 1000

Section	1	2	3	4	5
Vertical Load	11500	11500	11500	11500	11500
Horizontal	100	100	100	100	100
Vertical Displacement	1.00	1.00	1.00	1.00	1.00
Horizontal	1.00	1.00	1.00	1.00	1.00
Temperature	1.00	1.00	1.00	1.00	1.00
Total	11500	11500	11500	11500	11500
Factor	1.0	1.0	1.0	1.0	1.0
Reactions					
Vertical	11500	11500	11500	11500	11500

Section	1	2	3	4
Vertical Load	11500	11500	11500	11500
Horizontal	100	100	100	100
Vertical Displacement	1.00	1.00	1.00	1.00
Horizontal	1.00	1.00	1.00	1.00
Temperature	1.00	1.00	1.00	1.00
Total	11500	11500	11500	11500
Factor	1.0	1.0	1.0	1.0
Vertical Displacement	1.00	1.00	1.00	1.00

Note: The reactions are given in the table above.  
 The reactions are given in the table above.













Table of Ordinates to Parabolic - 40 Foot Span Table 7

Panel Point	0	1	2	3	4	5
Ordinate	0	2.5	4.0	5.2	6.0	6.5

Panel Point	0	1	2	3	4	5
Panel Point	0	1	2	3	4	5

0	0	0	0	0	0	0
1	2.5	4.0	5.2	6.0	6.5	
2	4.0	5.2	6.0	6.5	6.5	
3	5.2	6.0	6.5	6.5	6.5	
4	6.0	6.5	6.5	6.5	6.5	
5	6.5	6.5	6.5	6.5	6.5	

Table of Loads - 40 Foot Span

Panel Point	0	1	2	3	4	5
Vertical DL	0	1500	2000	2500	3000	3500
Horizontal DL	0	3000	4000	4500	4800	5000
Horizontal DL	0	3000	4000	4500	4800	5000
Horizontal DL	0	3000	4000	4500	4800	5000

Table of Shear - 40 Foot Span

Between panel points

0	1	2	3	4	5
0	1500	2000	2500	3000	3500
1	1500	1000	500	0	500

Max DL Negation

LL do

Temperature do

Total Vert Shear 0 1500 1000 500 0 500 1000

Required 1500 1000 500 0 500 1000

Required 1500 1000 500 0 500 1000

Load 0 1 2 3 4 5

Live 1 2 3 4 5

Dead 1 2 3 4 5

Dead 1 2 3 4 5

Live 1 2 3 4 5



Vertical Add

~~Homework~~

	6	4	5	5	100
5	1 2345 + 2345 = 4690	1 2345	4690		
6	4 5678 + 4567 = 9135	4 5678	9135		9135
7	5 6789 + 5678 = 11367	5 6789	11367		11367
8	6 7890 + 6789 = 13679	6 7890	13679		13679
9	7 8901 + 7890 = 15791	7 8901	15791		15791
10	8 9012 + 8901 = 17913	8 9012	17913		17913
11	9 0123 + 9012 = 18035	9 0123	18035		18035
12	10 1234 + 10123 = 20357	10 1234	20357		20357
13	11 2345 + 11234 = 23579	11 2345	23579		23579
14	12 3456 + 12345 = 25701	12 3456	25701		25701
15	13 4567 + 13456 = 27823	13 4567	27823		27823
16	14 5678 + 14567 = 29945	14 5678	29945		29945
17	15 6789 + 15678 = 32067	15 6789	32067		32067
18	16 7890 + 16789 = 34189	16 7890	34189		34189
19	17 8901 + 17890 = 36311	17 8901	36311		36311
20	18 9012 + 18901 = 38433	18 9012	38433		38433
21	19 0123 + 19012 = 40555	19 0123	40555		40555
22	20 1234 + 20123 = 42677	20 1234	42677		42677
23	21 2345 + 21234 = 44799	21 2345	44799		44799
24	22 3456 + 22345 = 46921	22 3456	46921		46921
25	23 4567 + 23456 = 49043	23 4567	49043		49043
26	24 5678 + 24567 = 51165	24 5678	51165		51165
27	25 6789 + 25678 = 53287	25 6789	53287		53287
28	26 7890 + 26789 = 55409	26 7890	55409		55409
29	27 8901 + 27890 = 57531	27 8901	57531		57531
30	28 9012 + 28901 = 59653	28 9012	59653		59653
31	29 0123 + 29012 = 61775	29 0123	61775		61775
32	30 1234 + 30123 = 63897	30 1234	63897		63897
33	31 2345 + 31234 = 66019	31 2345	66019		66019
34	32 3456 + 32345 = 68141	32 3456	68141		68141
35	33 4567 + 33456 = 70263	33 4567	70263		70263
36	34 5678 + 34567 = 72385	34 5678	72385		72385
37	35 6789 + 35678 = 74507	35 6789	74507		74507
38	36 7890 + 36789 = 76629	36 7890	76629		76629
39	37 8901 + 37890 = 78751	37 8901	78751		78751
40	38 9012 + 38901 = 80873	38 9012	80873		80873

Vertical Subtract

1	5 - 3 = 2	5	2	
2	6 - 4 = 2	6	2	
3	7 - 5 = 2	7	2	
4	8 - 6 = 2	8	2	
5	9 - 7 = 2	9	2	
6	10 - 8 = 2	10	2	
7	11 - 9 = 2	11	2	
8	12 - 10 = 2	12	2	
9	13 - 11 = 2	13	2	
10	14 - 12 = 2	14	2	
11	15 - 13 = 2	15	2	
12	16 - 14 = 2	16	2	
13	17 - 15 = 2	17	2	
14	18 - 16 = 2	18	2	
15	19 - 17 = 2	19	2	
16	20 - 18 = 2	20	2	
17	21 - 19 = 2	21	2	
18	22 - 20 = 2	22	2	
19	23 - 21 = 2	23	2	
20	24 - 22 = 2	24	2	
21	25 - 23 = 2	25	2	
22	26 - 24 = 2	26	2	
23	27 - 25 = 2	27	2	
24	28 - 26 = 2	28	2	
25	29 - 27 = 2	29	2	
26	30 - 28 = 2	30	2	
27	31 - 29 = 2	31	2	
28	32 - 30 = 2	32	2	
29	33 - 31 = 2	33	2	
30	34 - 32 = 2	34	2	
31	35 - 33 = 2	35	2	
32	36 - 34 = 2	36	2	
33	37 - 35 = 2	37	2	
34	38 - 36 = 2	38	2	
35	39 - 37 = 2	39	2	
36	40 - 38 = 2	40	2	
37	41 - 39 = 2	41	2	
38	42 - 40 = 2	42	2	
39	43 - 41 = 2	43	2	
40	44 - 42 = 2	44	2	

Vertical Multiply

1	1 x 1 = 1	1	1	
2	2 x 2 = 4	2	4	
3	3 x 3 = 9	3	9	
4	4 x 4 = 16	4	16	
5	5 x 5 = 25	5	25	
6	6 x 6 = 36	6	36	
7	7 x 7 = 49	7	49	
8	8 x 8 = 64	8	64	
9	9 x 9 = 81	9	81	
10	10 x 10 = 100	10	100	
11	11 x 11 = 121	11	121	
12	12 x 12 = 144	12	144	
13	13 x 13 = 169	13	169	
14	14 x 14 = 196	14	196	
15	15 x 15 = 225	15	225	
16	16 x 16 = 256	16	256	
17	17 x 17 = 289	17	289	
18	18 x 18 = 324	18	324	
19	19 x 19 = 361	19	361	
20	20 x 20 = 400	20	400	
21	21 x 21 = 441	21	441	
22	22 x 22 = 484	22	484	
23	23 x 23 = 529	23	529	
24	24 x 24 = 576	24	576	
25	25 x 25 = 625	25	625	
26	26 x 26 = 676	26	676	
27	27 x 27 = 729	27	729	
28	28 x 28 = 784	28	784	
29	29 x 29 = 841	29	841	
30	30 x 30 = 900	30	900	
31	31 x 31 = 961	31	961	
32	32 x 32 = 1024	32	1024	
33	33 x 33 = 1089	33	1089	
34	34 x 34 = 1156	34	1156	
35	35 x 35 = 1225	35	1225	
36	36 x 36 = 1296	36	1296	
37	37 x 37 = 1369	37	1369	
38	38 x 38 = 1444	38	1444	
39	39 x 39 = 1521	39	1521	
40	40 x 40 = 1600	40	1600	







Table of Bonding Moments -  
Horizontal Leads

4000 ft. or less

Table

Alt.  
Lead

	0	1	2	3	4
3	1 12.0	" 12.0	" 12.0	" 12.0	" 12.0
5	1 20.0	" 20.0	" 20.0	" 20.0	" 20.0
7	1 28.0	" 28.0	" 28.0	" 28.0	" 28.0
9	1 36.0	" 36.0	" 36.0	" 36.0	" 36.0
11	"	"	"	"	"
13	" 20.0	" 20.0	" 20.0	" 20.0	" 20.0
15	" 28.0	" 28.0	" 28.0	" 28.0	" 28.0
17	" 36.0	" 36.0	" 36.0	" 36.0	" 36.0
19	" 44.0	" 44.0	" 44.0	" 44.0	" 44.0
21	" 52.0	" 52.0	" 52.0	" 52.0	" 52.0

